



We’re headed out on some new adventures. We’re going to be tracking down more involved math mysteries. Are you ready for the challenge?



Here’s what you need to know. Wesley was 10 years old when his sister Leslie was born. With each passing year on the calendar, Wesley is still 10 years older than his sister because everyone ages one year at a time. We can compare this to the whole numbers where each number is ONE MORE than the previous one. That never changes. We can make a chart of their ages that shows this sequence.

	Leslie Born	In 1 year	In 2 years	In 3 years	In 4 years	In 5 years
Wesley	10	11	12	13	14	15
Leslie	0	1	2	3	4	5

No matter how many years we extend the chart, Wesley will always be 10 years older than Leslie.

What if we ask, “How many times older is Wesley than Leslie?” This question has to do with a different *relationship* between their ages. Unlike the difference between their ages, this relationship can be different over time. Let’s see how this works.

Look at column 3 in the chart. When Wesley is 12, Leslie is 2. Twelve divided by two is six. So after two years, Wesley will be 6 times older than Leslie.

Now look at column 6. When Wesley is 15, Leslie is 5. Fifteen divided by five is three. So after five years, Wesley will be only 3 times older than Leslie. This relationship between their ages changes over time, but the difference between their ages stays the same.



**Math Mystery #1:** *In how many years will Wesley be twice as old as his sister?*

We told you we would provide tracking tools to help you track the answers. Here’s the first one.



**EXTEND THE CHART**

Go ahead and extend the chart one year at a time until you find the answer. 

By continuing the chart, you can see that while the difference between their ages does not change, the number of times we multiply Leslie’s age to equal Wesley’s age does change. In the 11th column we see that Wesley is 20 and Leslie is 10.  $20 \div 10 = 2$ . **So after ten years when Wesley is 20, Leslie will be 10, and Wesley will be twice as old as his sister.**

We were able to track down the answer by making a chart of their ages. But what if we had to track down the following mystery?

**Math Mystery #2:** *John is 36 years older than his daughter Marie. How old will Marie be when John is twice as old as his daughter?* Using a chart to find the answer will take a lot of work. Making a chart is not always an efficient way to solve the problem. Remember, in math we want to be efficient as well as correct.

This is why developing your reasoning skills is so helpful. Here's another tracking tool you already know, plus three new tools.



MAKE A DRAWING THAT SHOWS WHAT YOU KNOW; THIS REPRESENTS YOUR STARTING POINT.



SHOW THE SMALLEST AMOUNT WITH A BLOCK. SHOW ALL OTHER AMOUNTS BASED ON THAT BLOCK.



MAKE A DRAWING THAT SHOWS WHAT YOU NEED TO FIND OR WHERE YOU WANT TO END UP.

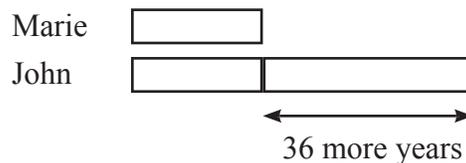


PUT THE TWO DRAWINGS TOGETHER TO HELP YOU REASON THROUGH THE PROBLEM.

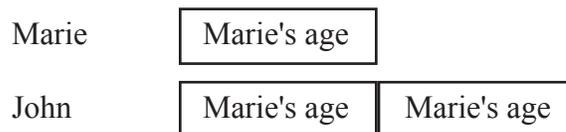
Let's track down this mystery together. First, what do we know? John is 36 years older than Marie. We draw a short bar that represents Marie's age (because her age is the smallest). Then we draw a longer bar that represents John's age.

➤ Do you remember from the age problems you have already done that if a person is older we add more years to the drawing and if a person is younger we subtract years from the drawing?

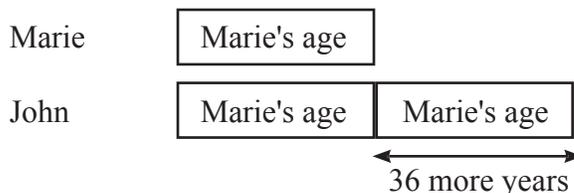
The drawings don't need to be exact; you only need to make a rough estimate to picture the information. We mark off the part of John's bar that shows he is 36 years older. This drawing is our starting point.



The drawing below shows the relationship of their ages. We want to show their ages when John is twice as old as Marie. So we make a bar that represents Marie's age and then put two bars of the same size together (twice Marie's age) to represent John's age.



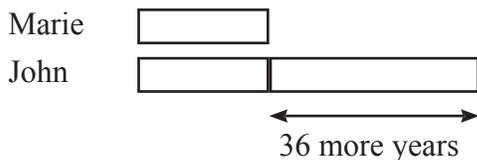
Now let's combine both pictures to track our way through all the information. This drawing shows both facts: (1) John is twice as old as Marie; and (2) John is 36 years older than Marie.



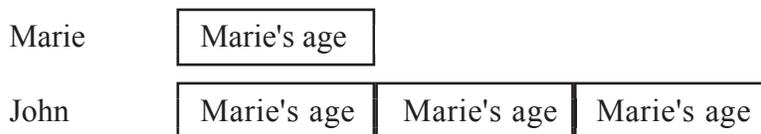
Now we can see that each of the bars representing John’s age equals 36 years.  $36 + 36 = 72$  years. We can also see that each bar is the same as Marie’s age, which means her bar equals 36 years. We’ve tracked down the answer. **When Marie is 36, John will be 72, which is twice as old as his daughter.** We can check our work by plugging the numbers back into the story.  $36 \times 2 = 72$ . Check.

**Math Mystery #3:** Now let’s track down when John will be three times older than his daughter.

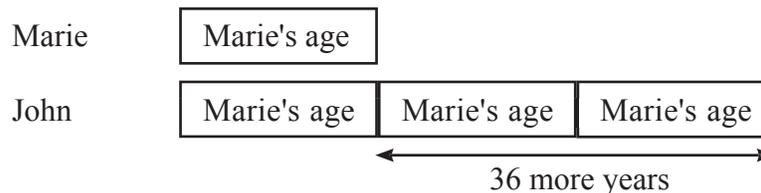
We can use the first drawing again as our starting point since the difference in their ages is always the same.



This time the relationship picture needs to show John’s age as 3 times Marie’s age.



We then combine the first and second drawings to show the pathway to our destination.



This helps us see how the relationship in their ages changed. We can see that 2 times Marie’s age must equal 36 years. Remember, the relationship as to how many times one age must be multiplied to equal the other age can change over time, but the difference in their ages never changes.

When we divide 36 in half, we can find what each bar represents, which will show us Marie’s age.

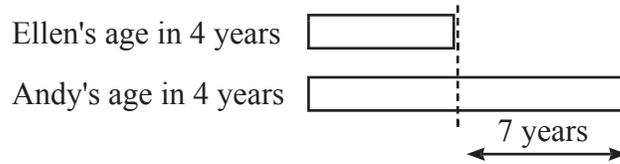
$$36 \div 2 = 18. \text{ Marie is 18 years old}$$

Since each bar represents 18 years, we can multiply by 3 to find John’s age.  $3 \times 18 = 54$ . We’ve tracked down the answer. **When Marie is 18, John will be 54, which is 3 times his daughter’s age.**

- Can you see that in both examples, the difference between John’s age and his daughter’s age stayed the same (36 years)?
- Can you see that the number of times we multiplied Marie’s age to equal John’s age did change? That relationship between their ages changes over the years.

**Math Mystery #4:** *Andy is 7 years older than his sister Ellen. In 4 years Andy will be twice as old as his sister. How old are they now?*

This time we have a new situation to consider: “in 4 years.” Apart from that time factor, the problem is the same as the other mysteries we’ve tracked down. Since Andy’s age is twice Ellen’s age in 4 years we will make our drawing showing their ages in 4 years at which time Andy will still be 7 years older than Ellen.



Now we can see that the block showing one-half of Andy's age equals 7 years. So Andy will be  $2 \times 7 = 14$  and Ellen will be 7. The story tells us that this relationship (Andy being twice as old) happens in 4 years. So to find their ages now, we simply subtract 4 years from their ages.

**Ellen's age now:  $7 - 4 = 3$  years. Andy's age now:  $14 - 4 = 10$  years.**

In problems like these that include either a time in the future or a time in the past, we find the ages that show the relationship described in the problem first. Then we either subtract (when the time period described is in the future) or we add (when the time period described is in the past).

### Practice

Track down these math mysteries. Be sure to show all three drawings as well as the arithmetic. Check your work by plugging the numbers back into the story. Don't forget the units.

- (1) When John is four times Marie's age, how old will each of them be? (This is the same John and Marie from the examples.)
- (2) Alice is 12 years older than Tim. What are their ages when Alice is twice as old as Tim?
- (3) Find their ages when Alice is three times as old as Tim.
- (4) Find their ages when Alice is four times as old as Tim.
- (5) Mr. Connors is 26 years older than his son Lyle. Find their ages when Mr. Connors is three times as old as his son.