In the last lesson we reviewed:

- ✓ the definition of a ratio;
- ✓ the three ways to write a ratio; and
- ✓ how ratios can be used to compare various items as long as we are careful with the order.

Now that we know what the ratio tool looks like, we can see how to use it and uncover its real power. Let's take the example we used in the last lesson to another level: *In a book reading contest, Craig and his friend Brian read a total of 40 books. The ratio of the books Craig read to the books Brian read is 5 to 3. How many books did each boy read?*

Understanding what a ratio describes helps us solve this type of problem. Remember, **it describes a relationship, not a quantity**. The given ratio is the key. "The ratio of the books Craig read to the books Brian read is 5 to 3."

- ✓ The ratio does NOT tell us that Craig read 5 books and Brian read 3 books (quantity).
- ✓ What the ratio DOES tell us is that for every 5 books Craig read, Brian read 3 books (relationship).

We also know the total number of books read: 40. Imagine taking the 40 books read and returning 5 of the books to Craig and 3 of the books to Brian over and over in that same manner until all 40 books have been returned to the proper person. The table below shows what happens.

relationship: returning 5 of the books Craig read and 3 of the books Brian read

round	**books Craig now has**	**books Brian now has**	**total books**
1	5	3	8
2	5 + 5 = 10	3 + 3 = 6	16
3	5 + 10 = 15	3 + 6 = 9	24
4	5 + 15 = 20	3 + 9 = 12	32
5	5 + 20 = 25	3 + 12 = 15	40

Now we know that when all 40 books are handed out in the ratio of 5 to 3, Craig has 25 books and Brian has 15 books. That means our answer is Craig read 25 of the 40 books total and Brian read 15 of the 40 books total. The chart shows us that ratios can be used to show the relationship between:

- PARTS – Craig's books to Brian's books = 5 to 3
- PARTS and the WHOLE – Craig's books to the total books = 5 to 8; Brian's books to the total books = 3 to 8.

Notice that the totals in the chart in Craig's column are multiples of 5, the totals in Brian's column are multiples of 3, and the numbers in the Total Books column are multiples of 8. These are multiples of 8 because in each round 5 + 3 = 8 books were handed out.

Now let's express these relationships using the three forms for writing ratios.

relationship: returning 5 of the books Craig read and 3 of the books Brian read

round	books Craig now has	books Brian now has	word form	fraction form	symbol form
1	5	3	5 to 3	$^5/_3$	5 : 3
2	10	6	10 to 6	$^{10}/_6$	10 : 6
3	15	9	15 to 9	$^{15}/_9$	15 : 9
4	20	12	20 to 12	$^{20}/_{12}$	20 : 12
5	25	15	25 to 15	$^{25}/_{15}$	25 : 15

Notice that the fractions comparing Craig's books to Brian's books in each round are all **equivalent fractions**.

round 1 = $^5/_3 \div {}^1/_1 = {}^5/_3$
round 2 = $^{10}/_6 \div {}^2/_2 = {}^5/_3$
round 3 = $^{15}/_9 \div {}^3/_3 = {}^5/_3$
round 4 = $^{20}/_{12} \div {}^4/_4 = {}^5/_3$
round 5 = $^{25}/_{15} \div {}^5/_5 = {}^5/_3$

> Remember, two fractions are equivalent if you can multiply or divide one of the fractions by a form of 1 and get the other fraction.

There is no end to the number of equivalent fractions you can find for any fraction you are given because our numbers keep going on and on indefinitely (or infinitely). However, only one of these possible fractions can be said to be in lowest terms. Remember, a fraction is in lowest terms if no whole number other than 1 will divide evenly into both the numerator and the denominator. In this case, $^5/_3$ is in lowest terms.

Since fractions are another way to write a ratio, we can also say these are **equivalent ratios**. And only one of these ratios will be in lowest terms. Just as we can multiply or divide both the numerator and denominator of a fraction by the same number to find an equivalent fraction, we can multiply or divide both the first and second terms in a ratio by the same number to find an equivalent ratio.

round 1 = 5 x 1 = 5; 3 x 1 = 3 **5 : 3**
round 2 = 5 x 2 = 10; 3 x 2 = 6 **10 : 6**
round 3 = 5 x 3 = 15; 3 x 3 = 9 **15 : 9**
round 4 = 5 x 4 = 20; 3 x 4 = 12 **20 : 12**
round 5 = 5 x 5 = 25; 3 x 5 = 15 **25 : 15**

We also can find the simplest form of a ratio just as we can find the lowest terms of a fraction. For example, to find the simplest form of the ratio 5 : 15, we divide both terms of the ratio by the greatest common factor, which is 5. The simplest form is 1 : 3.

When we are working with a ratio, each round shows the same relationship. When we were giving the total of 40 books back to each boy, we had to hand them out round by round in the same ratio of 5 to 3. Each round is called a **distribution**. You can think of a distribution as "dealing out" the total number of items in a set way—the specified ratio.

Let's suppose Alyssa and Matt want to play two card games. The rules of the first game say they must deal out the cards in a ratio of 2 : 1. The rules of the second game say they must deal out the cards in a ratio of 4 : 2. Both games are played with 18 cards.

Game 1- ratio is 2 : 1(18 cards)

round	Alyssa	Matt	total dealt
1	2	1	3
2	2 + 2 = 4	1 + 1 = 2	6
3	2 + 4 = 6	1 + 2 = 3	9
4	2 + 6 = 8	1 + 3 = 4	12
5	2 + 8 = 10	1 + 4 = 5	15
6	2 + 10 = **12**	1 + 5 = **6**	**18**

Game 2 – ratio is 4 : 2 (18 cards)

round	Alyssa	Matt	total dealt
1	4	2	6
2	4 + 4 = 8	2 + 2 = 4	12
3	4 + 8 = **12**	2 + 4 = **6**	**18**

You can see that no matter which ratio was used, Alyssa ended up with 12 cards and Matt ended up with 6 cards. Why? Because the ratios of 2 : 1 and 4 : 2 are equivalent ratios. In game 1 it took 6 rounds or distributions to deal the 18 cards, while in game 2 it only took 3 rounds to deal the 18 cards.

Notice, too, that in each game the distribution remained the same in each round. In each round of game 1, Alyssa always got 2 more cards and Matt always got 1 more card. In each round of game 2, Alyssa always got 4 more cards and Matt always got 2 more cards. In game 2 the cards were given out twice as fast so it is reasonable that all 18 cards were dealt out twice as fast (3 rounds in game 2 compared to 6 rounds in game 1).

Based on this example only, you might think it is always better to use the ratio with the largest numbers because you get to the answer faster. However, this is not always the case. This is another example of being careful not to rush to a conclusion. It is almost always better to work with a ratio that is in lowest terms (simplest form). You'll see why in the lessons that follow.

Practice

Write YES if the pair of fractions is equivalent and NO if it is not.

(1) $^2/_3$ and $^{10}/_{15}$ (2) $^1/_4$ and $^3/_8$ (3) $^{18}/_2$ and $^{54}/_6$ (4) $^{14}/_1$ and $^{140}/_7$

Write three equivalent fractions for each of the following fractions.

(5) $^4/_2$ (6) $^3/_5$ (7) $^1/_6$

Reduce the following fractions to their lowest term.

(8) $^{15}/_{25}$ (9) $^{12}/_8$ (10) $^{22}/_{55}$

Write YES if the pair of ratios is equivalent and NO if it is not.

(11) 4 : 1 and 28 : 5 (12) 2 : 5 and 8 : 15 (13) 9 : 3 and 27 : 9

Write two equivalent ratios for each of the following ratios.

(14) 7 : 3 (15) 6 : 9 (16) 10 : 2

Reduce the following ratios to their lowest term.

(17) 48 : 8 (18) 5 : 50 (19) 56 : 42

Find the answers. *There are 18 girls and 24 boys in the running club.*

(20a) What is the ratio of the number of girls to the number of boys?

(20b) What is the ratio of the number of boys to the total number of children?

(20c) What is the ratio of the number of girls to the number of children?

(20d) Write the above three ratios in lowest terms.